Uniform Acceleration

July 21, 2019

Uniform acceleration by Susskind's "hyperbola" method

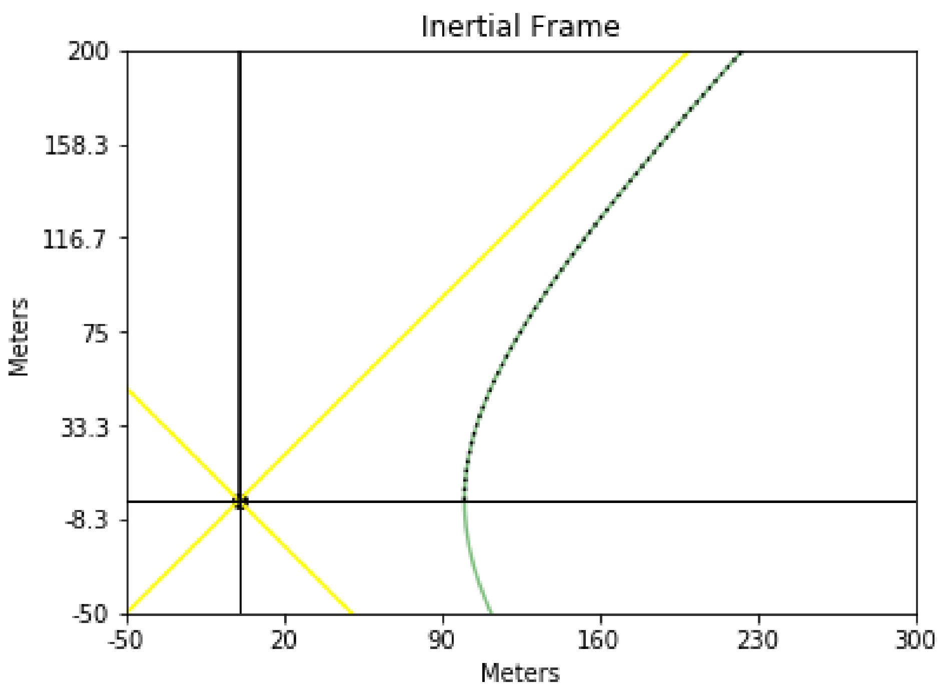
> Let r be the desired acceleration.
> $x = r \cosh \omega$
> $t = r \sinh \omega$
> As we vary ω this draws a hyperbola with its vertex at r.

Numeric Acceleration

$$a' = a(1 - v^2)^{\frac{3}{2}}$$

> Given a and v in the accelerating frame, a' will be the acceleration in an inertial frame. This can be used to plot the curve of an accelerating object as seen from the inertial frame.

Below I've drawn the hyperbola in green and the corresponding acceleration via the numeric formula with black dots. (The yellow lines are the light cone through the origin.) The hyperbola and the standard acceleration formula agree. So far, so good :-)



This is the metric tensor for uniform acceleration. We have not derived this. I got it from the Susskind lecture I mentioned earlier.

$$g_{\mu\nu} = \begin{bmatrix} -(2Gx+1) & 0 \\ 0 & 1 \end{bmatrix}, \quad g^{\mu\nu} = \begin{bmatrix} \frac{-1}{2Gx+1} & 0 \\ 0 & 1 \end{bmatrix}$$

From the metric tensor we derive Γ, which turns out to have three non-zero components:

$$\Gamma^t{}_{tx} = \frac{G}{2Gx+1}$$

$$\Gamma^t{}_{xt} = \frac{G}{2Gx+1}$$

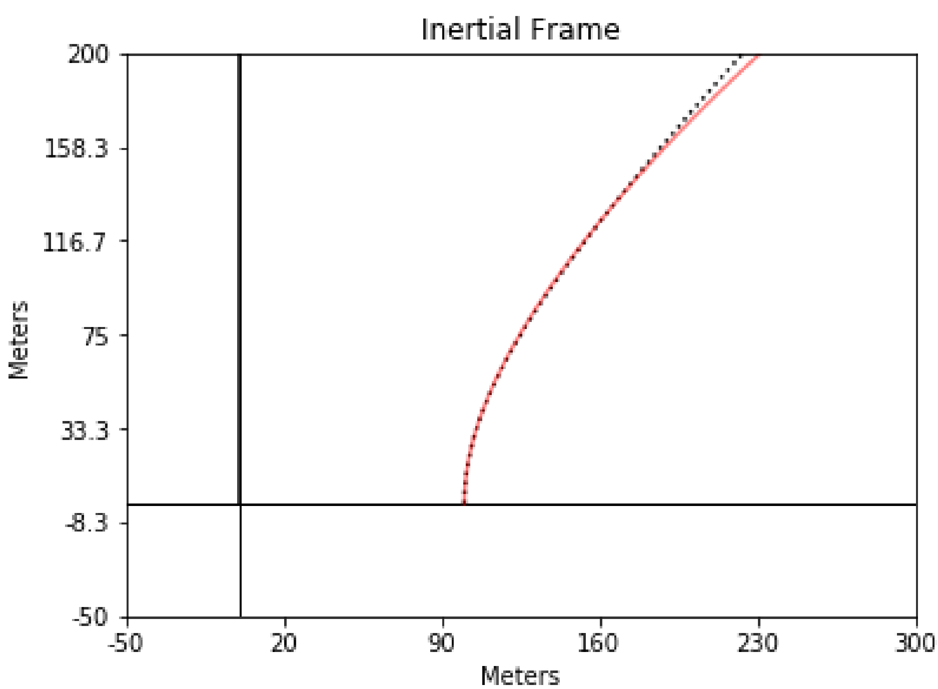
$$\Gamma^x{}_{tt} = G$$

Then, working out the geodesic equation, we get:

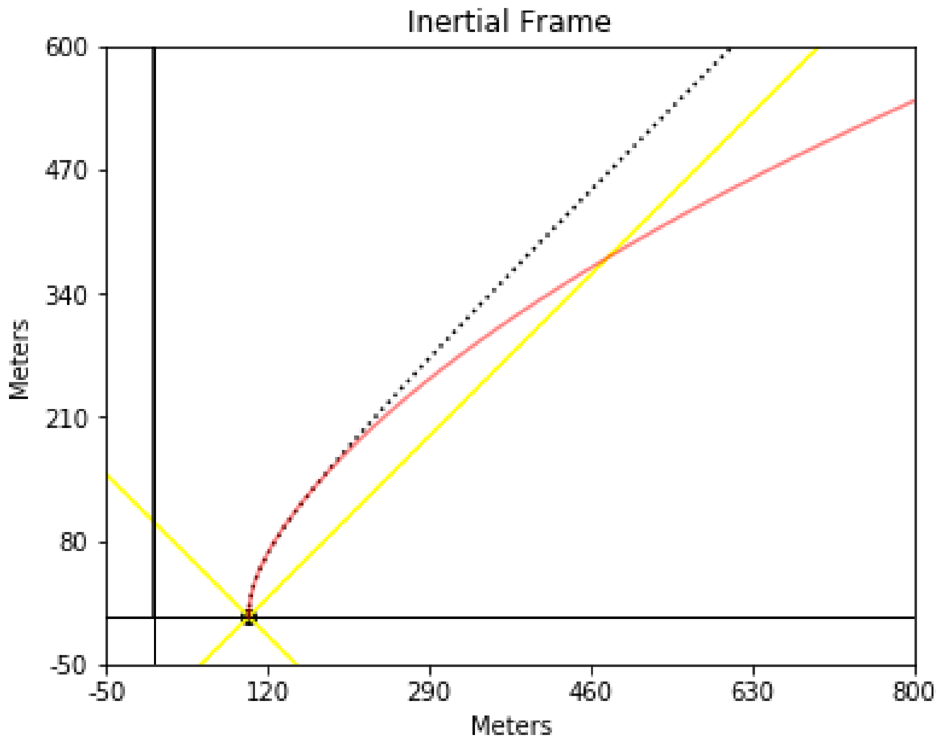
$$\frac{d^2t}{d\tau^2} = -\frac{2G}{2Gx+1}\frac{dt}{d\tau}\frac{dx}{d\tau}$$

$$\frac{d^2x}{d\tau^2} = -G\left(\frac{dt}{d\tau}\right)^2$$

Then we look at what happens as we vary the speed and position of the object. Below the black dots represent the path of the accelerated object using the same acceleration formula from above. The pink line is what I get by plotting the path using the geodesic equation.



As you can see, they agree when the velocity is low, but then the pink path starts going faster than the dotted one. It looks like something is wrong here. In fact, if I run out the plot a little further, my geo-calculated line ends up crossing the light cone. Not good!



As far as I can tell, the calculation of the geodesic equation is correct. So what I think at this point is that either (1) I've got some conceptual problem in the way I'm applying it, or (2) There's just a bug in my code somewhere. I'm guessing it's probably (1).